How to Teach the Bar Model Method

What are bar models in maths?

Bar models are pictorial representations of problems or concepts that can be used for any of the operations: addition, subtraction, multiplication and division. In word problems, they hold the huge benefit of helping children decide which operations to use or visualise problems. It is then up to the children to calculate the answer.

5 Step Guide to Bar Models

1) Concrete resources (real objects) *1:1 representation*

2) Substituted concrete resources (counters, cubes, buttons) *1:1 representation*

3) Pictorial representations (circles) *1:1 representation*

4) Objects as part of a bar (individual squares) *1:1 representation*

5) Rectangular bars (approximates)

ADDITION

Example question for addition: *Children will routinely come across calculations such as 3 + 2.*
Often, these calculations will be presented as word problems:
Marissa has 3 apples. Lucas has 2 apples.   How many apples are there altogether?

Using Concrete Resources (1)

With addition, subtraction and multiplication, to help children fully understand the later stages of bar modelling, it is crucial they begin with concrete representations. There are 2 models that can be used:

?

Marissa's apples   **Lucas' apples**

or

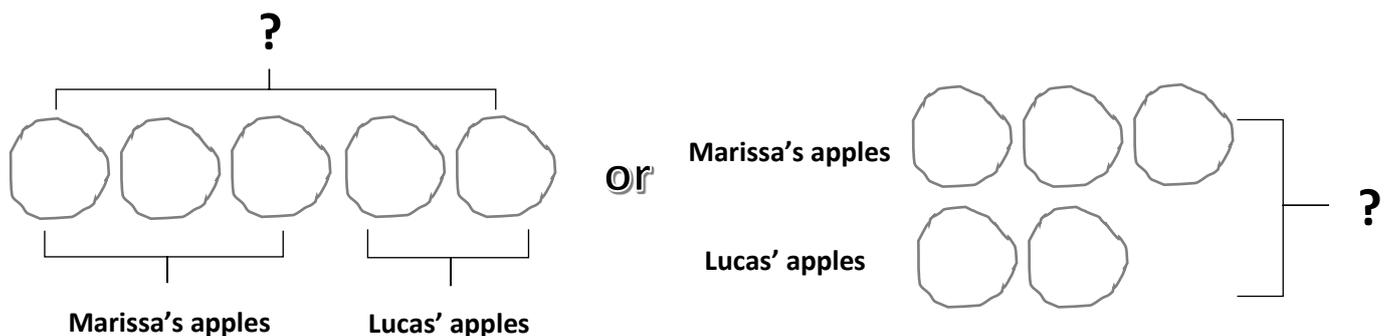**Marissa's apples**

Lucas' apples

?
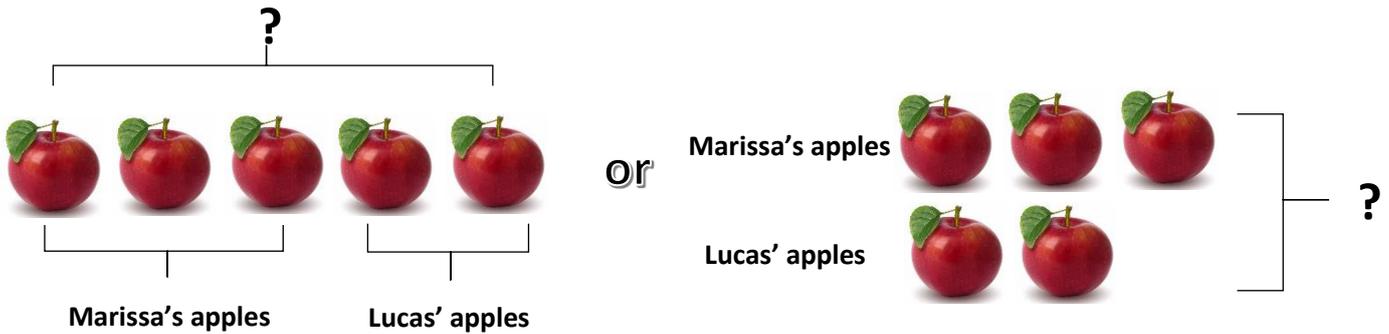
Using Substituted Concrete Resources (2)

Once they are used to the format and able to represent problems in this way themselves (assigning 'labels' verbally), the next stage is to replace the 'real' objects with objects that represent the object.

?

Marissa's apples   **Lucas' apples**

or

Marissa's apples

Lucas' apples

?
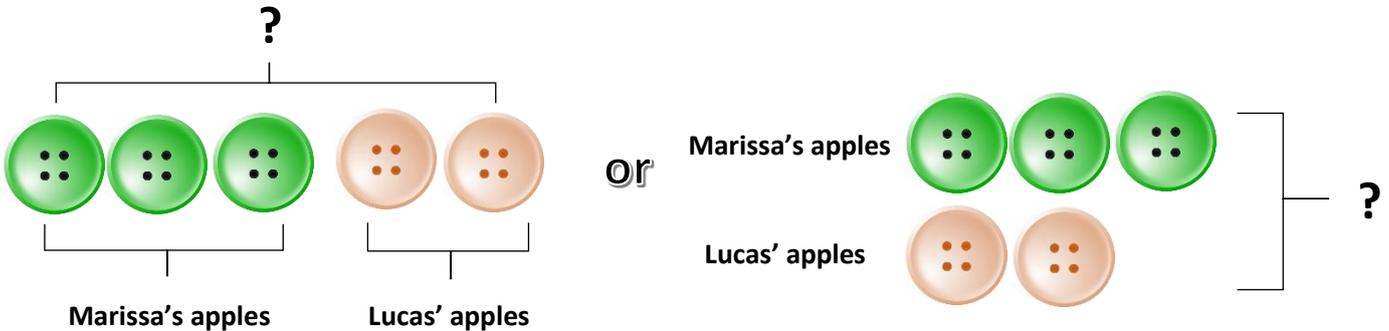
Pictorial Representations (3)

The next stage is to move away from the concrete to the pictorial.

?

Marissa's apples   **Lucas' apples**

or

Marissa's apples

Lucas' apples

?

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Objects as Part of a Bar (4)

The penultimate stage is to represent each object as part of a bar, in preparation for the final stage:

?

Marissa's apples   **Lucas' apples**

or

Marissa's apples

Lucas' apples

?
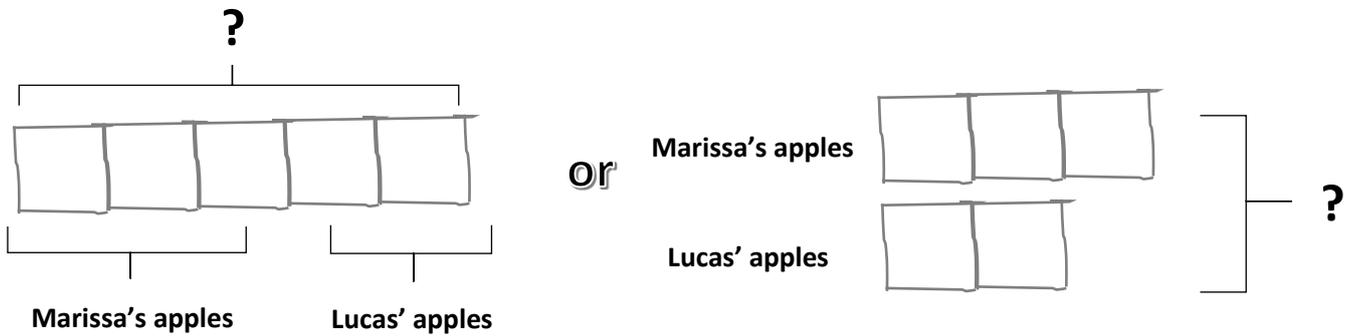
Rectangular Bars (5)

The final stage stops the 1:1 representation and instead, each quantity is represented approximately as a rectangular bar.

?	
3	2

Marissa's apples   **Lucas' apples**

or

Marissa's apples | 3 |

**Lucas' apples** | 2 |

?

SUBTRACTION

The same concrete to pictorial stages can be applied to subtraction.
However, whereas with addition it is really down to the pupil's preference as to which of the 2 bar representations to use. One represents a 'part-part-whole' model, whereas the other is a 'find the difference' model. Each will be more suited to different word problems and different pupils.

Lucas has 15 playing cards. He gives 9 to his sister. How many cards does he have left?

15	
9	?

or

Lucas has 15 playing cards. Marissa has 6 playing cards. How many more playing cards does Lucas have than Marissa?

Lucas' cards | 15 |

Marissa's cards | 6 |

?

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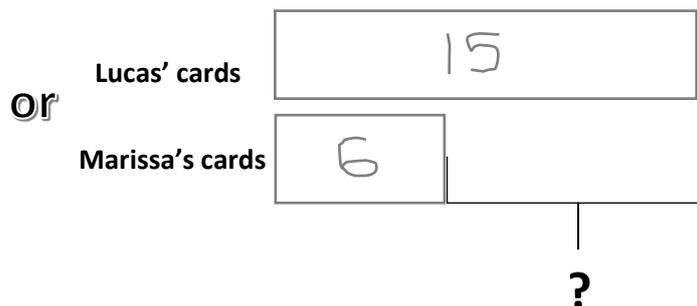
MULTIPLICATION

Bar models of multiplication start with the same 'real' and 'substituted' concrete resource stages as addition and subtraction.
Then moves to its final stage, drawing rectangular bars to represent each group:

Example question for multiplication: *Children will routinely come across calculations such as 5 x 4. Often, these calculations will be presented as word problems:*
Marissa buys four boxes of cookies. Each box contains 5 cookies. How many cookies does Marissa have?

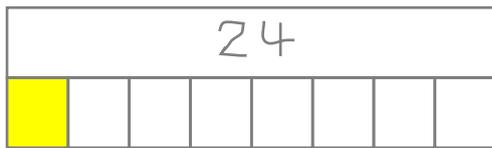| ?			
5	5	5	5
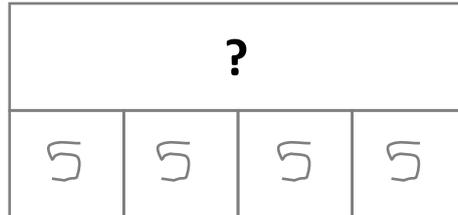
DIVISION

Due to the complexity of division, it is recommended to remain grouping and sharing until the final stage of bar modelling is understood. Then word problems such as the 2 below can be introduced:
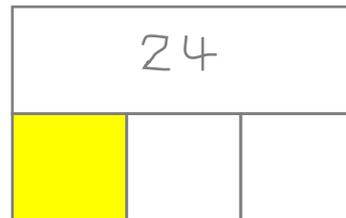
Sharing

Lucas has 24 lollies. He wants to share them into 8 party bags for his friends. How many lollies will go into each party bag?
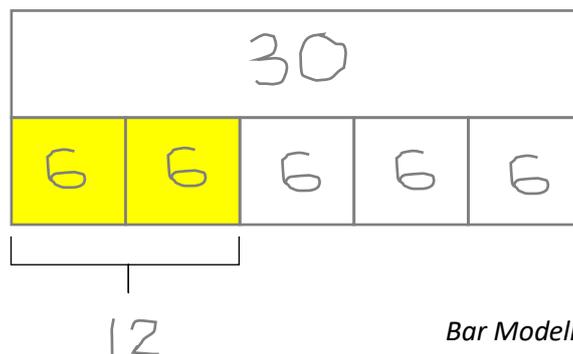
24							

Grouping

Lucas has 24 lollies for his party friends. He wants each friend to have 3 lollies. How many friends can he invite to his party?

24		

FRACTIONS

It is beneficial for children to be taught the relationship between fractions (and percentages) and division, such as ⅕ is the same as ÷5
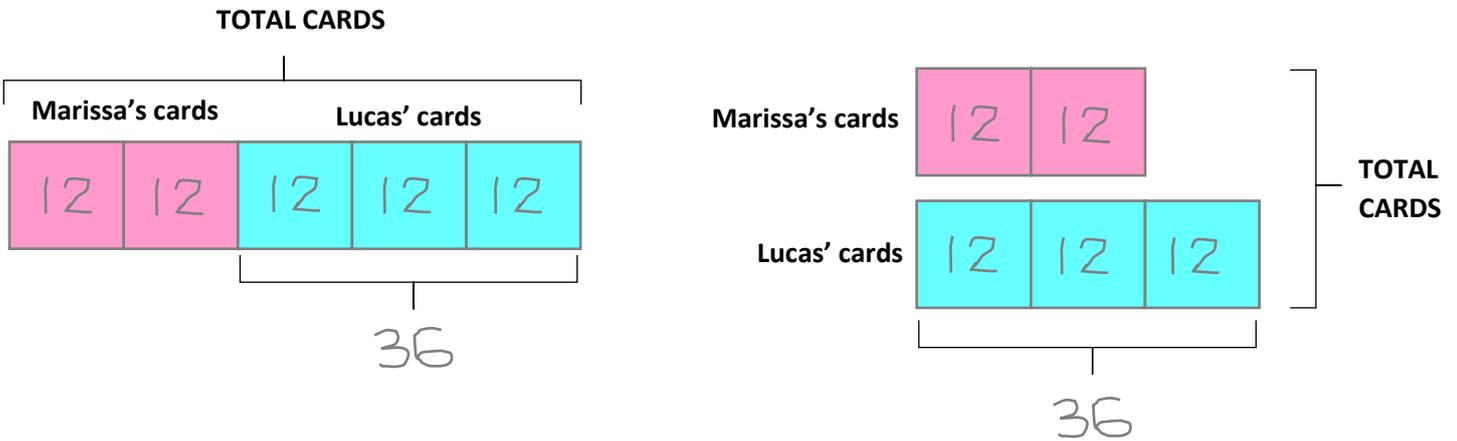
What is 2/5 of 30?

30				
6	6	6	6	6

12

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RATIO

Ratio can be linked to fractions and division and can be solved in two ways:

Marissa and Lucas share some Lego cards in the ratio of 2:3. If Lucas has 36 Lego cards, how many cards are there altogether?

TOTAL CARDS

Marissa's cards   **Lucas' cards**

| 12 | 12 | 12 | 12 | 12 |

36

Marissa's cards | 12 | 12 |

Lucas' cards | 12 | 12 | 12 |

TOTAL CARDS

36

This method can also be used when scaling up or down:

Marissa and Lucas have £100 between them. Marissa has three times more than Lucas. How much money does Marissa have?

Marissa's money   Lucas' money

£100

Marissa's money

Lucas' money

£100
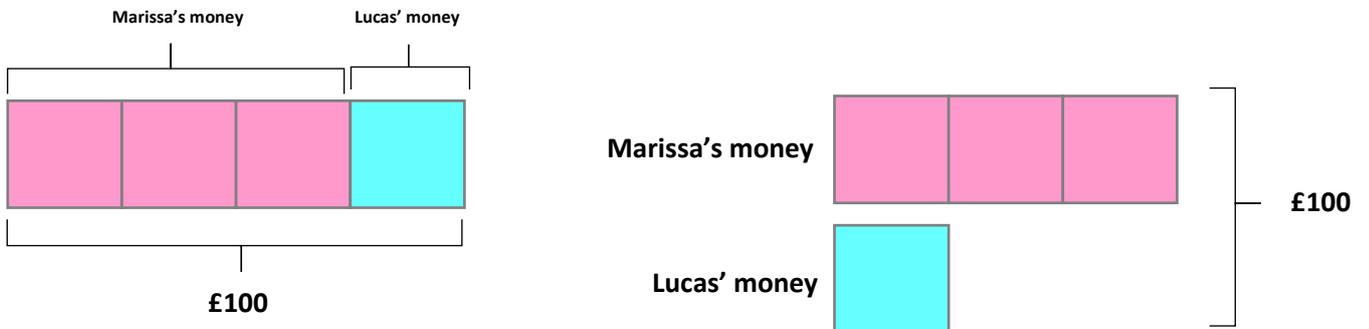
UNKNOWN VALUES

When solving questions with unknown values

Marissa and Lucas have £100 between them. Marissa has £20 more than Lucas. How much money does Marissa have?

Marissa's money   £40   £20

Lucas' money   £40

£100   £80

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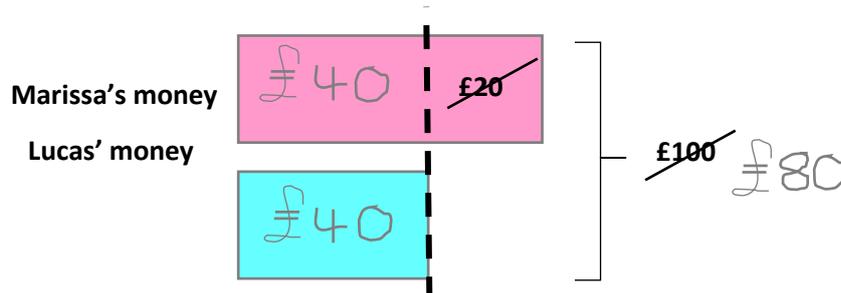
ALGEBRA

Algebraic equations can be solved using bar models using the strategy above and division:

3a + 5 = 17

What is the value of a?

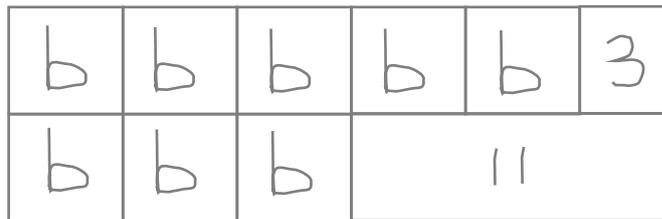| 12 ~~17~~			5
a	a	a	5
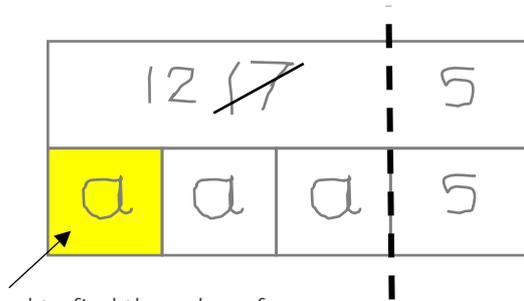
Division can then be used to find the value of a

Algebraic equations with unknowns on both sides of the equation:

**5b + 3 = 3b + 11**

What is the value of b?

b	b	b	b	b	3
b	b	b		11	

Division can then be used to find the value of b

b	b	3
8 ~~11~~		3
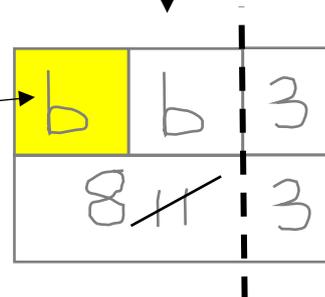
MULTI-STEP

Multi-step problems can be solved using a combination
of bar models using two or more operations.

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Primary School

Five friends are having a pizza party. They buy one large pizza and three small pizzas. They share the cost equally. How much does each person pay?

Small pizza £4

Large pizza £6

1 large pizza   **3 small pizzas**

| £6 | £4 | £4 | £4 |

← **Total cost**

Individual cost
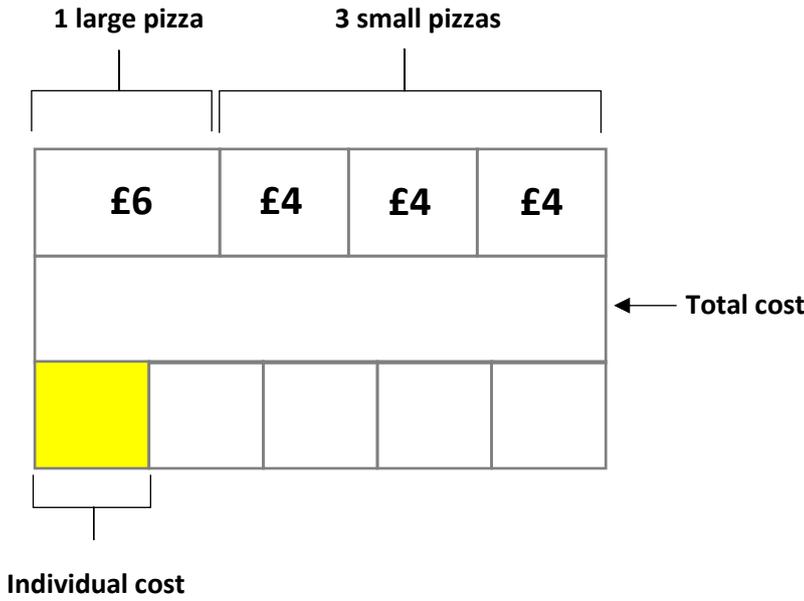
Many other mathematical problems can be solved using bar models.

ELSTON HALL
Primary School

When shall I teach bar modelling?

Basic Concepts

Bar models can be used to show basic concepts, e.g. *9 + 7*

16	
9	7

Therefore, bar models can be used just to introduce children to bar models without them actually using them but simply to display the question.

This is will allow children to give numbers a value/size visually and see their relative size and help to compare numbers.
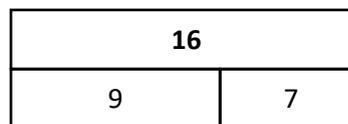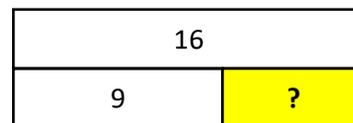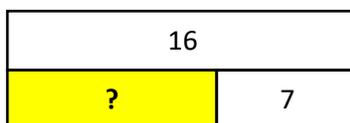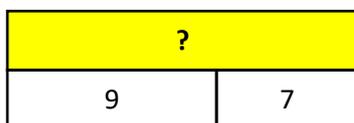
Variation

They can also be used as procedural exercises for variation:

9 + 7 = _____   16 – 7 = _____   16 – 9 = _____

?	
9	7

16	
?	7

16	
9	?

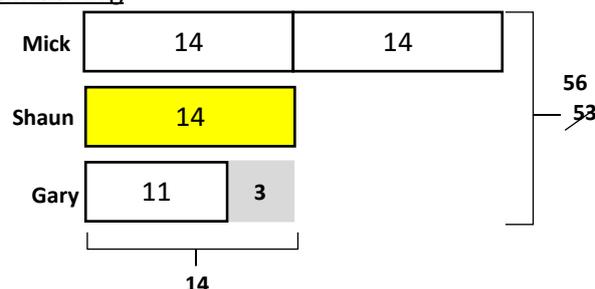### Problem Solving

Finally, bar models can be used as a problem solving tool,

e.g.   *Mick, Shaun and Gary are brothers.*
Mick is twice as old as Shaun.
Gary is 3 years younger than Shaun.
The sum of all their ages is 53.
How old is Shaun?

Mick | 14 | 14 |
Shaun | 14 |
Gary | 11 | 3 |
56
53
14

Bar modelling is an extremely useful visual tool to help children understand number problems. It is not the intention to replace our existing calculation policy but to slot alongside it to provide additional support. Bar modelling is for everyone: children of all abilities and adults too.

Use bar modelling to reinforce understanding.

MATHEMATICS is not about numbers, equations, computations, or algorithms: it is about **UNDERSTANDING.**   So let's help children understand.

William Paul Thurston

Concrete – students should have the opportunity to use concrete objects and manipulatives to help them understand what they are doing.

Pictorial – students should then build on this concrete approach by using pictorial representations. These representations can then be used to reason and solve problems.

Abstract – with the foundations firmly laid, students should be able to move to an abstract approach using numbers and key concepts with confidence.